Problem 1
A gantry can be approximated as a linear system when the swing angles are small. Such a model, with the payload position $X$ as the output and the force to the trolley $F$ as the input, is:

$$\frac{X}{F} = \frac{g}{M L s^4 + M g s^2}$$

Use the parameters $M=10\text{kg}$, $L=6\text{m}$, and $g=10\text{m/s}^2$.

A. Use MATLAB to plot the first 30 seconds of the response of this model to several bang-bang inputs. Use bang durations ranging from $t_2=2.5$ to 8 seconds at steps of 0.5 seconds. The bang-bang function is shown in Figure 2 and is given by:

$$U(t) = 1(t) - 2 * 1(t-t_2) + 1(t-t_3)$$

B. Design an input shaper for the system using the damped period, $T_d$ and the damping ratio, $\zeta$. The formulas for a ZV shaper impulses are:

$$A_i = \begin{bmatrix} 1 & K \\ 0 & 0.5T_d \end{bmatrix} \quad \text{where, } K = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

C. Convolve the bang-bang commands in part A with the input shaper in part B to create shaped input commands. Plot the response on the system to these shaped commands.

Problem 2
Consider the two-mass-spring example considered in class. Assume there is only input acting on the first mass. Assume that only the position of the first mass can be measured. Design an observer to predict the system states.

Problem 3
Reconsider the system in Problem 2. Assume the position of the first mass cannot be measured, but the position of the second mass can be measured. Design a reduced-order observer for the system.