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SLOSH SUPPRESSION OF A LIQUID IN A SUSPENDED CONTAINER USING ROBUST INPUT SHAPING

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Controlling oscillations induced by the motion of a suspended liquid container is a problem for many industries. For example, when overhead cranes move containers of molten metal the sloshing of the liquid can be dangerous and degrade the metal processing operation. Traditional feedback controllers require liquid-motion sensors that are difficult to implement. Moving the container with commands generated by the input-shaping method is an alternative that does not require sensors. Input shaping has the potential to reduce both residual and transient peak slosh amplitudes, while accommodating change in slosh frequency. It has already been proven that input shaping is effective in reducing liquid slosh, however no experimental data exists for moving liquid containers suspended by overhead cables. This paper describes simulation and experimental results demonstrating that robust input shaping is very effective at reducing peak transient and residual oscillation in cable-suspended liquid containers, even when the model parameters are not exact.

1. Introduction

Moving a liquid-filled container from one location to another poses a problem in many applications because the liquid can slosh and spill. For example when moving molten metal, spillage could be costly and dangerous. When the container is suspended from an overhead crane, the natural frequencies of both the crane and the liquid inside the container are excited. The aim of this paper is to demonstrate how input-shaping techniques can be used to reduce both residual and transient slosh when moving a liquid-filled container suspended from a crane. Experiments were conducted on a portable bridge crane. Various input shapers were examined for their performance in reducing liquid sloshing and robustness to parameter variation.

2. System Modeling

To simulate the system's dynamic response to various input-shaped commands, linear models of the payload dynamics and liquid-sloshing dynamics were created. The payload, a rigid container with liquid inside, was constrained to prohibit rotation. During experimental testing the payload was moved only in the trolley direction to produce planar motion. A schematic drawing of the suspended payload is provided in Figure 1, where *xt* is the motion of the trolley, *L* is the suspension length measured to the middle of the water level, Θ is the deflection angle of the suspension cable, and *y* is the horizontal displacement of the container. Each mass-spring-damper in the system model represents a vibratory mode of liquid slosh.

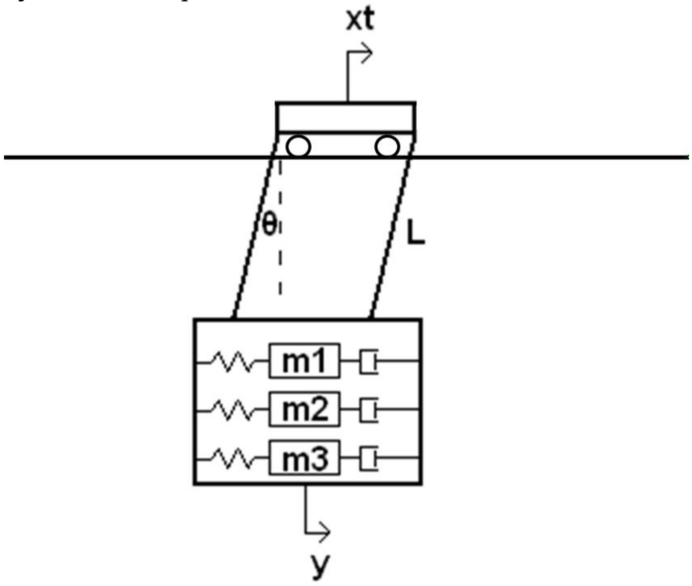

Figure 1. Schematic Drawing of Suspended Liquid Container

2.1 Slosh Dynamics

The dynamics of sloshing liquid inside a container have been studied by various researchers[1,2,3,4,11]. However, the previous work did not study the problem using an overhead crane as the container mover. The pendulum frequency of the suspended container is ω_o. A liquid's 1-dimensional sloshing motion can be modeled as a system of damped harmonic oscillators[4] as shown in Figure 1. Each independent mass-spring-damper system corresponds to a quantity of liquid that oscillates at one of the fundamental slosh frequencies, which are given by[4]:

$$\omega_i^2 = (2i-1)\frac{g\pi}{a}\tanh\left((2i-1)\frac{h\pi}{a}\right) \tag{1}$$

where ω_i represents the i^{th} sloshing frequency (i = 1, 2,...), a is the container length, h is the container depth, and g is the acceleration due to gravity. The natural frequency of the suspended payload and the frequencies corresponding to each liquid slosh mode will be called ω_o, ω_1, ω_2, etc. throughout the paper. Analytical expressions have been developed for the damping ratio, ζ, showing it to be approximately 0.01 for water[4]. Table 1. Nominal Dynamic System Parametersshows the frequencies obtained for a particular liquid depth of 42 mm in the container and suspension length of 740 mm.

Table 1. Nominal Dynamic System Parameters

L (mm)	ω_o (rad/s)	a (mm)	h (mm)	ζ	i	ω_i (rad/s)
740	0.579	250	42	0.01	1	1.229
					2	2.934
					3	3.931

To evaluate the performance of different input shapers, the individual dynamic models of trolley position, payload position, payload velocity and slosh dynamics were combined in a Simulink model. The output response of the suspended container forms the input to the three mass-spring-damper systems corresponding to the first three vibration modes of liquid slosh. The slosh amplitudes of each vibration mode are scaled and summed to form the overall sloshing response.

2.2 Input Shaping

To tackle the problem of container pendulum oscillation and liquid slosh, a method called input shaping was used. Input shaping convolves the velocity command with a series of impulses at specific time instances to form a specially-shaped command that reduces residual vibration[5,6]. Convolving multiple input shapers together to suppress multiple modes results in higher rise time, but the benefit is that residual vibration will be reduced in a broader range of frequencies and the command will be more robust to modeling errors and parameter variations

The input-shapers used in this paper include: Zero Vibration (ZV) Shaper[5,6], Zero Vibration and Derivative (ZVD) [5,6], EI-Extra Insensitive (EI) Shaper [7,8].

2.3 Simulation Results

The simulations predicted that the liquid slosh could be significantly reduced, but not eliminated, by input shaping the pulse velocity commands to the trolley. The transient slosh associated with the actual move is unavoidable (because the system must accelerate and necessarily compress the liquid “spring”) and can only be reduced by moving slower. Input shaping does slow the system down slightly to accomplish this goal somewhat. More importantly, the residual vibration after the move can be reduced to a great extent by using a combination of input shapers.

The simulation predicted that any combination of shapers that suppresses vibration at ω_o and ω_1 will effectively reduce the residual vibration. Suppressing vibration at additional sloshing modes (2nd, 3rd, etc.) was shown to have minor benefit while rendering the response slower. An example comparison of unshaped and input-shaped responses is provided in Figure 2. Suppressing the vibration of ω_o offers the largest reduction in vibration, reducing the transient response amplitude by about 50% and reducing the residual vibration by about 85% compared to the unshaped case. Adding an additional shaper to suppress vibration at ω_1 offers little additional benefit to the transient response, but almost completely eliminates residual slosh.

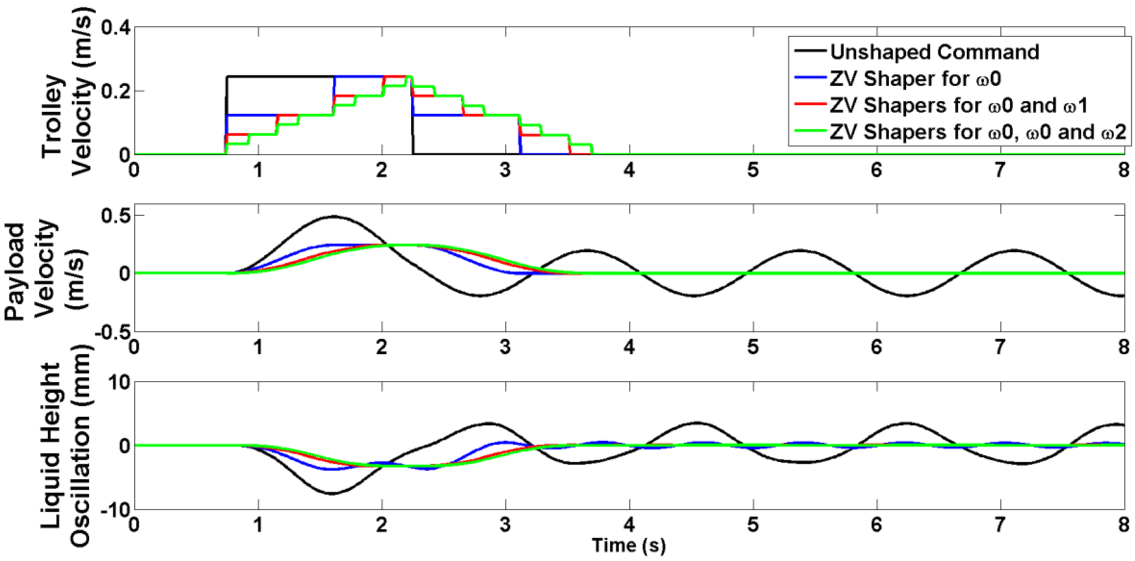

Figure 2. Simulation results

2.4 Robustness Simulations

The slosh response will always vary with move distance, liquid depth, and suspension length. Variation in these parameters was studied because there is uncertainty in the liquid depth and cable length. Therefore, the input shaper designed should be robust to modeling errors and variations in move distance.

2.4.1 Variation with Move Distance

Figure 3 shows the variation in the amplitude of liquid slosh vibration with move distance when the trolley is driven by an unshaped velocity commands for move distances between 0.2 - 0.8 meters. There is a complex relationship between move distance and the slosh amplitude. At a move distance of about 0.6 m, it can be observed that the amplitude of slosh is greatly reduced. This is because the vibration induced by the 'go' command interferes with the vibration induced by the 'stop' command so as to cancel each other out. Hence, the total vibration which is a resultant of the two interfering waves is almost zero. The opposite occurs when the move distance is about 0.4 m. Here, the two vibrations add up resulting in an amplitude of about 10 mm. When the input command was shaped using ZV Shapers for ω_0, ω_1 and ω_2, the amplitude of slosh vibration reduced significantly and there was little variation in amplitude with move distance, as seen in Figure 4.

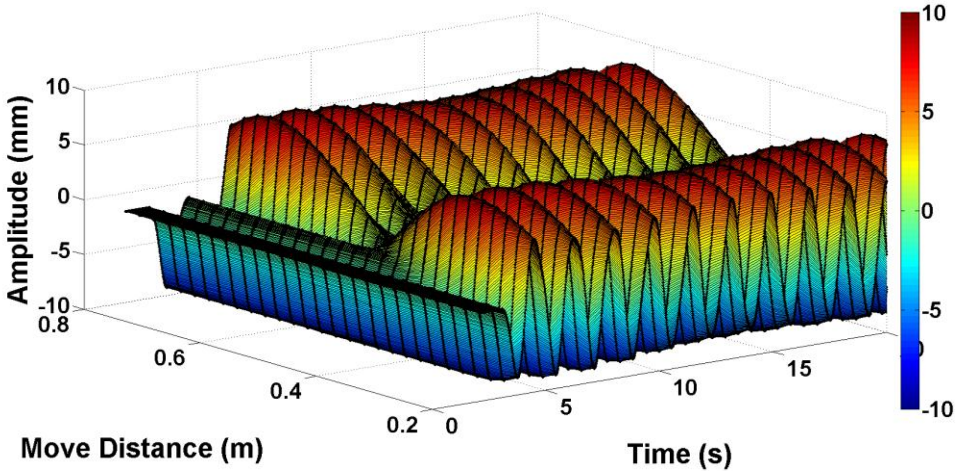

Figure 3. Variation in the Amplitude of Liquid Slosh with Move Distance using an Unshaped Velocity Profile, keeping the Liquid Depth at 42 mm and Suspension Length at 740 mm

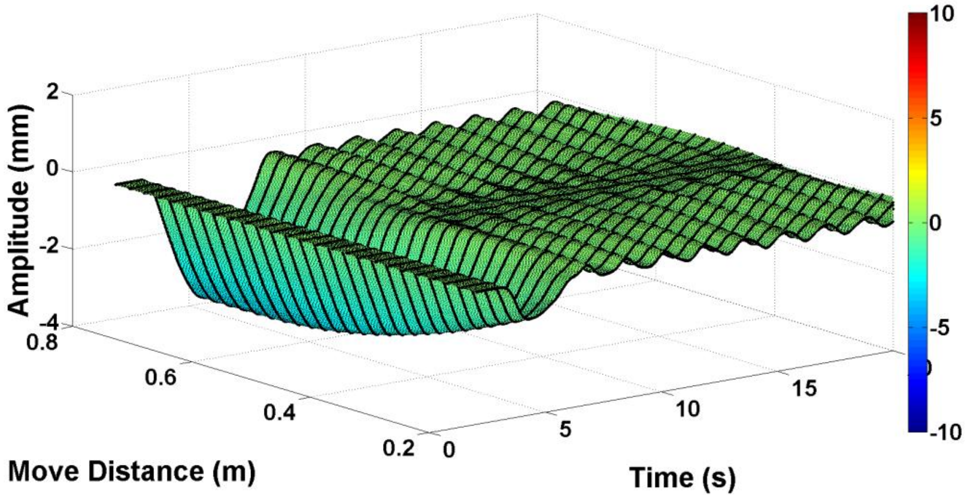

Figure 4. Variation in the Amplitude of Liquid Slosh with Move Distance using ZV shapers for ω_o, ω_1 and ω_2, keeping the Liquid Depth at 42 mm and Suspension Length at 740 mm

2.4.2 Variation with Liquid Depth

As the liquid depth changes, the sloshing frequencies of the liquid change in accordance with Eq. (1). Simulations showed that the slosh vibration amplitude is high for shallow liquid depths and reduces as the depth increases. The amplitude of slosh does not significantly after the liquid depth reaches a critical level.

A shaper designed to suppress oscillation of the liquid slosh modes must be robust to the wide range of frequencies that can occur as the liquid depth changes. Using a ZV shaper for the pendulum mode and a 5% EI shaper for ω_1, the vibration was reduced over a wide range of liquid depths.

2.4.3 Variation with Suspension Length

As the length of the suspension cable is varied, ω_o varies. This variation in the pendulum swing causes a variation of the liquid slosh amplitude as well, when moved by an unshaped velocity profile. However using two 5% EI shapers, one designed to suppress vibrations at ω_o at 740 mm, and the other to suppress vibrations at ω_1, significantly reduces the slosh over a wide range of suspension lengths.

3. Experiment

3.1 Experimental Setup

A container filled with dyed water was moved with a portable bridge crane[9] controlled with a Siemens PLC. The container was moved only along the trolley axis. To eliminate rotation of the container, four lines from each of the corners were attached to a fixture mounted to the trolley of the bridge crane, as can be seen in Figure .

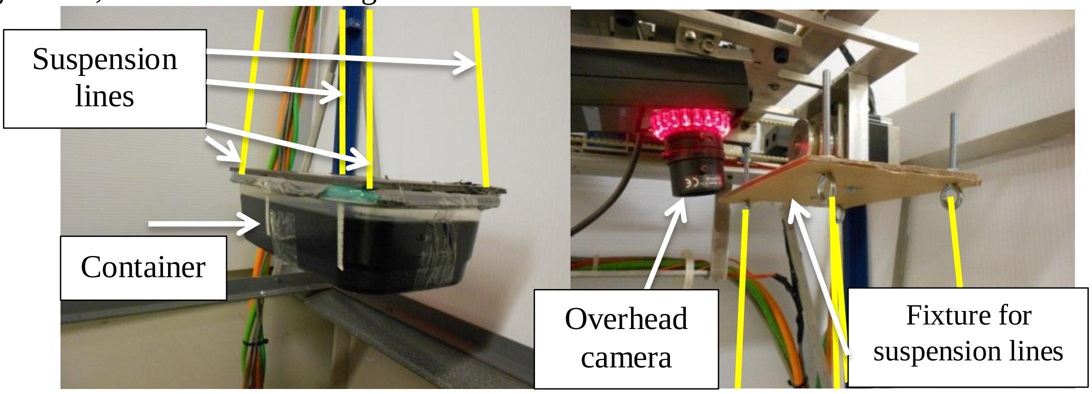

Figure 5. Experimental Setup

A video camera was moved along a beam parallel to the trolley axis for video analysis in MATLAB. Water slosh amplitude was measured by evaluating the vertical distance between the bottom of the container and the top left corner on the water surface, as shown in Figure 6. The distance was measured every frame in pixels and then converted to millimeters using reference markings on the container.

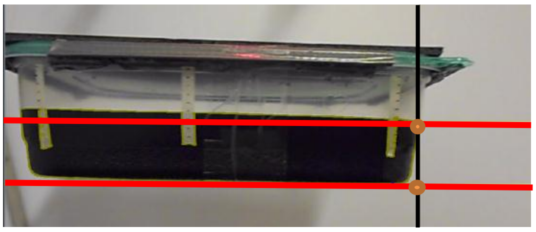

Figure 6. Measuring water level at the corner

3.2 Experimental verification of simulation data

Figure 7 shows a comparison of the simulation results and experimental results for an unshaped command and shaped command. It can be seen that the results are closely matched which shows that the model for the sloshing frequencies and choice of damping ratio of water was fairly accurate. The differences arise largely from higher-mode dynamics and measurement noise

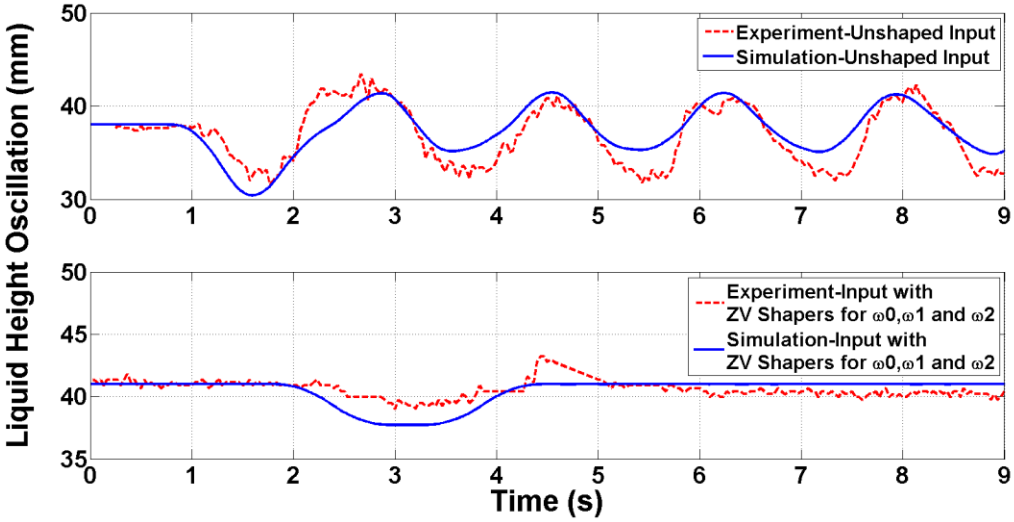

Figure 7. Comparison of Experimental and Simulation Data

3.3 Experiments

A summary of the results is tabulated in Table 2. From the table it can be seen that shaping for ω_o with a ZV shaper and for ω_1 with an EI shaper with 5% tolerance, produced the lowest residual peak slosh with a 92% improvement from the unshaped. This is very close to the response obtained with a ZV shaper for ω_o and ZVD shaper for ω_1. However, because it is also important to keep the transient slosh minimal to avoid spillage, other shapers that produce lower transient slosh are attractive alternatives. Using ZV Shapers designed to suppress vibrations at ω_o, ω_1 and ω_2, the peak residual slosh was reduced by 89% and the peak transient slosh was reduced by 80%.The additional benefit of the extra ZV shaper is not solely caused by canceling the inherent vibration of the 2nd liquid frequency (which is very low in amplitude), it also introduces additional lag in the system response and reduces the peak acceleration that makes the system slower. The sensitivity curves in Figure 8 illustrate why the system behaved as it did by showing how much the various shapers suppressed vibration at different frequencies.

Table 2. Experiment Results

(Suspension length =740 mm, Liquid depth=42 mm, Move Distance=0.3645 m)

Shaper Type			Peak Transient Slosh (% of unshaped)	Peak Residual Slosh (% of unshaped)
ω_0	ω_1	ω_2		
-	-	-	100	100
ZV	-	-	24	16
ZVD	-	-	33	14
5% EI	-	-	29	16
ZV	ZV	-	35	11
ZV	ZVD	-	35	9
ZV	5% EI	-	30	8
ZV	ZV	ZV	20	11

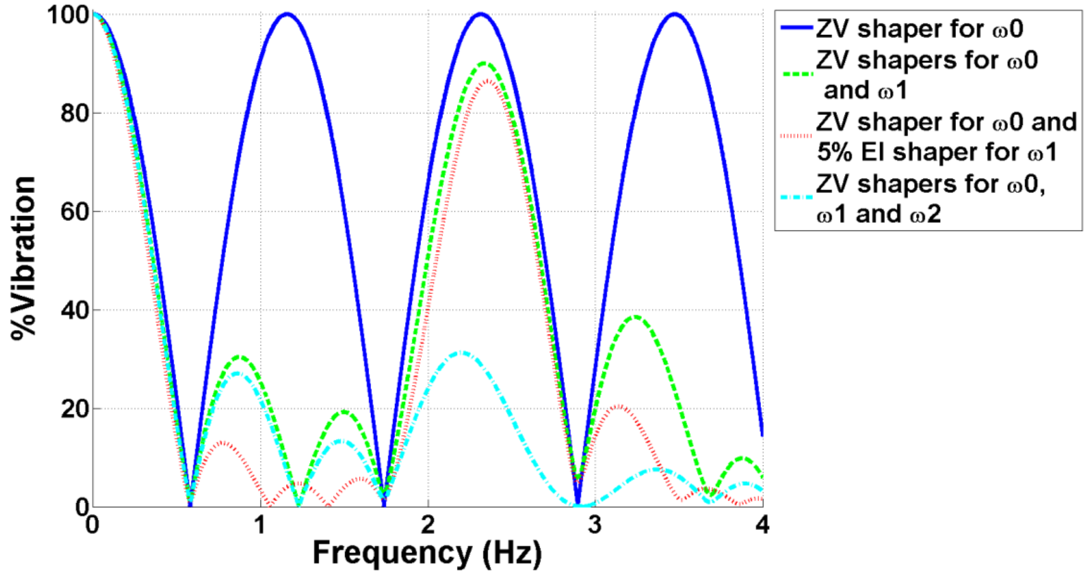

Figure 8. Sensitivity Plots for the Shapers used

3.4 Robustness tests

Because it was determined that the triple ZV and the ZV-EI combination produced the best slosh minimization results, the robustness of these two shapers was tested. The robustness to changes in move distance, liquid depth in the container, and suspension length of the container were tested. Table 3 shows the performance of the two shapers as each of the three parameters was varied. It can be seen that the triple ZV shaper and ZV-EI shapers reduce both transient and residual slosh about equally.

Table 3. Robustness to Move Distance, Liquid Depth and Suspension Length

Shaper Type			Move Distance (m)	Liquid Depth (mm)	Suspension Length (mm)	Peak Transient Slosh (% of unshaped)	Peak Residual Slosh (% of unshaped)
ω_0	ω_1	ω_2					
ZV	ZV	ZV	0.2430	42	740	22	9
ZV	ZV	ZV	0.3645	42	740	20	11
ZV	ZV	ZV	0.4860	42	740	34	14
ZV	ZV	ZV	0.3645	31	740	22	11
ZV	ZV	ZV	0.3645	64	740	40	11
ZV	ZV	ZV	0.3645	42	515	22	11
ZV	ZV	ZV	0.3645	42	625	34	11
ZV	5% EI	-	0.2430	42	740	16	9
ZV	5% EI	-	0.3645	42	740	30	8
ZV	5% EI	-	0.4860	42	740	30	14
ZV	5% EI	-	0.3645	31	740	24	14
ZV	5% EI	-	0.3645	64	740	49	14
ZV	5% EI	-	0.3645	42	515	32	11
ZV	5% EI	-	0.3645	42	625	34	9

4. Conclusion

Simulations and experiments showed that input shaping is well suited to reduce slosh in a suspended rectangular container. Shapers designed for this experiment were fairly robust to changes in move distance, liquid depth in the container, and suspension length of the container. Three-mode ZV and ZV- EI shapers performed the best amongst the shapers studied. The drawback of input shaping is that the shapers that suppress more oscillation also slow the system down more. Additional research is required into more robust and faster shapers, such as the Specified Insensitivity (SI) shaper which may give better results.

REFERENCES

[1] Takanori Miyoshi, Yoshiyuki Noda, and Kazuhiko Terashima. Feedforward Control Considering Input and States Constraints with Eliminating Residual Vibration, *Proceedings of the 2007 American Control Conference*, New York, NY, July 2007.

[2] Ameen Aboel-Hassan, Mustafa Arafa, and Ashraf Nassef. Design and optimization of input shapers for liquid slosh suppression, *Journal of Sound and Vibration*, **320**, 1–15, (2009).

[3] Pridgen, Brice, Kun Bai, William Singhose. Slosh Suppression by Robust Input Shaping, *IEEE Conference on Decision and Control*, Atlanta, USA, 2010.

[4] Pridgen, Brice, *Comparison Of Polynomial Profiles And Input Shaping For Industrial Applications*, Master of Science Thesis, George W. Woodruff School of Mechanical Engineering, (2011).

[5] Neil C. Singer and Warren P. Seering. Preshaping command inputs to reduce system vibration, *Journal of Dynamic Systems, Measurement, and Control*, **112**, 76–82, (1990).

[6] W Singhose. Command shaping for flexible systems: A review of the first 50 years, *International Journal of Precision Engineering and Manufacturing*, **10**(4), 153–168, (2009).

[7] Singhose, W., Seering, W. and Singer, N. Shaping inputs to reduce vibration: A vector diagram Approach, *IEEE Int. Conf on Robotics and Automation*, **2**, 922-927, Cincinnati, OH, (1990).

[8] W. Singhose, W. Seering, and N. Singer. Residual Vibration Reduction Using Vector Diagrams to Generate Shaped Inputs, *Journal of Mechanical Design*, **116**, 654-659, (1994).

[9] Jason Lawrence and William Singhose. Design of minicrane for education and research, *6th Int. Conference on Research and Education in Mechatronics*, Annecy, France, 2005.

[10] W. Singhose, W. Seering, and N. Singer. Input Shaping for Vibration Reduction with Specified Insensitivity to Modeling Errors, *Japan-USA Sym. on Flexible Automation*, Boston, MA, 1996.

[11] J. Feddema, C. Dohrmann, G. Parker, R. Robinett, V. Romero, and D. Schmitt, Control for Slosh-Free Motion of an Open Container, *IEEE Control Systems*, vol. 17, pp. 29-36, 1997.