Radial-Motion Assisted Command Shapers for Nonlinear Tower Crane Rotational Slewing

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Abstract

Input shaping is an effective method for reducing motion-induced vibration. The majority of input-shaping theory is based on linear analysis; however, input shaping has proven effective on moderately nonlinear systems. This work investigates the effect of nonlinear crane dynamics on the performance of input shaping. Typical bridge cranes are driven using Cartesian motions and behave nearly linearly. The rotational structure of a tower crane makes nonlinearities more apparent. Nonlinear equations of motion are presented and experimentally verified. Novel command-shaping algorithms are then proposed for reducing vibration during the nonlinear slewing motions of tower cranes

Key words: input shaping, crane, nonlinear dynamics, vibrations, rotational dynamics

1 Introduction

Input shaping is a common method for reducing motion-induced vibration, wherein the reference command is altered by convolving it with a series of impulses, as shown in Figure 1 (Smith, 1957; Singer and Seering, 1990; Singhose and Pao, 1997; Huey et al., 2008). Input shaping has been shown to be very effective on Cartesian robots and bridge cranes (Starr, 1985; Singer et al., 1997), even when
the payload undergoes moderate hoisting (Singhose et al., 2000). The majority of input-shaping theory is based on the impulse response of linear systems. The non-dimensional residual vibration induced by a series of impulses applied to an underdamped oscillator is:

\[
V = e^{-\zeta \omega t_n} \sqrt{[C(\omega, \zeta)]^2 + [S(\omega, \zeta)]^2}
\]

where,

\[
C(\omega, \zeta) = \sum_{i=1}^{n} A_i e^{\zeta \omega t_i} \cos(\omega \sqrt{1 - \zeta^2 t_i}), \quad S(\omega, \zeta) = \sum_{i=1}^{n} A_i e^{\zeta \omega t_i} \sin(\omega \sqrt{1 - \zeta^2 t_i})
\]

and \(\omega\) is the natural frequency of the system, \(\zeta\) is the damping ratio, and \(A_i\) and \(t_i\) are the amplitudes and time locations of the impulses, respectively (Singer and Seering, 1990). If a series of impulses are to produce zero residual vibration, then the resulting series of impulses is called a Zero Vibration (ZV) input shaper (Smith, 1958; Singer and Seering, 1990). For undamped systems, the ZV shaper is given by:

\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
1/2 \\ 1/2 \\
0 \\ T/2
\end{bmatrix}, \quad i = 1, 2
\]

where \(T\) is the period of oscillation being suppressed.

While the ZV shaper will produce zero vibration when the dynamic properties \((\omega, \zeta)\) and know exactly, there will be some residual vibration when modeling errors exist. To add robustness to modeling errors, the derivative of vibration with respect to frequency can also be forced to zero at the modeled frequency. An impulse sequence satisfying this additional constraint is called a Zero Vibration and Derivative (ZVD) shaper (Singer and Seering, 1990). The ZVD shaper for undamped systems is given by:

\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
1/4 \\ 1/2 \\ 1/4 \\
0 \\ T/2 \\ T
\end{bmatrix}, \quad i = 1, 2, 3
\]

These input shapers can be convolved with any reference command to yield a shaped command with vibration-reducing properties. The above shapers only target one vibration mode; however, multi-mode vibration suppression is also easily accomplished with input shaping (Hyde and Seering, 1991; Singhose et al., 1997b; Lim et al., 1999; Kim and Singhose, 2007; Singhose et al., 2008).

Cranes are ideal candidates for input shaping because they typically exhibit only one or two modes of vibration and they are lightly damped; so any induced oscillation will not be quickly dissipated. However, because input shaping relies on superposition to cancel vibration and the dynamics of all cranes are somewhat non-linear, the reliance on superposition is not appropriate for all operating regimes of all cranes.
This paper investigates tower crane dynamics and control. Nonlinear equations of motion are presented and then verified using a portable tower crane at the Tokyo Institute of Technology (Lawrence et al., 2006). The nonlinear behavior of the tower crane is examined, as well as its effect on input shaping. It is shown that for tower cranes, nonlinear dynamics can be significant. Novel command-shaping algorithms are then proposed for dealing with the nonlinear tower crane dynamics. While the results of this analysis are developed for tower cranes, they can also be extended to other types of cranes such as boom cranes (Parker et al., 1999; Arnold et al., 2003; Maleki and Singhose, 2009) and quayside cranes (Bosnjak et al., 2006; Zrnic et al., 2005; Hong and Hong, 2004).

While feedback controllers have also been used to control cranes with single and multiple frequency dynamics (Tanaka and Kouno, 1998; Kim et al., 2004; Kenison and Singhose, 2002; Sawodny et al., 2002; Abdel-Rahman et al., 2003; Sorensen et al., 2007), they are often difficult and expensive to install and maintain. Additionally, there can be difficulties in using feedback control in conjunction with human operators because the automatic feedback control induces motions that the human operator cannot always anticipate. Therefore, this paper focuses on command-shaping control methods.

2 Tower Crane Dynamics

2.1 System and Modeling

An illustration of a tower crane is shown in Figure 2. The trolley moves along the jib in the radial, \( R \), direction, and the jib rotates around the mast in the \( s \) direction. If the suspension length is constant, then the equations of motion relating the payload swing angles, \( \phi \) and \( \theta \), to the motion of the trolley in the \( R \) and \( s \) directions are:

\[
L \ddot{\phi} + L \dot{\theta}^2 \cos(\phi) \sin(\phi) + g \sin(\phi) \cos(\theta) = -\dot{R} \cos(\phi) + R \dot{s}^2 \cos(\phi) \\
-R \dot{s} \sin(\phi) \sin(\theta) - 2R \dot{s} \sin(\phi) \sin(\theta) - 2L \dot{s} \theta \cos^2(\phi) \cos(\theta) \\
-2L \dot{s} \sin(\theta) + L \dot{s}^2 \sin(\phi) \cos^2(\theta) \cos(\phi) = 0
\]

(5)

\[
L \ddot{\theta} \cos(\phi) - 2L \dot{\phi} \theta \sin(\phi) + g \sin(\theta) = R \ddot{s} \cos(\theta) + 2R \dot{s} \cos(\theta) \\
+2L \dot{s} \phi \cos(\phi) \cos(\theta) + L \dot{s} \sin(\phi) \cos(\theta) + L \dot{s}^2 \sin(\phi) \cos(\phi) \cos(\theta)
\]

These can be partially linearized to:

\[
\ddot{\phi} = -\frac{g}{L} \phi - \frac{1}{L} \dot{R}, \quad \ddot{\theta} = -\frac{g}{L} \theta + \frac{R}{L} \dot{s}
\]

(6)
These equations are not entirely linear, as demonstrated by the $R/L$ term. A nominal radius, $R_0$, could be used to completely linearize the equations. This is not done here so that the dynamics can be investigated over a large range of $R$.

2.2 Model Verification

The natural motion of a rotating tower crane is an arc. This leads to nonlinear behavior in the payload’s movement. Therefore, tower cranes exhibit noticeable nonlinear behavior for motions that include moderate-to-high-speed rotational components. Figure 3 shows the responses for a combined rotational slew and radial move. Both the full nonlinear simulation and experimentally measured responses are shown. As can be seen, the nonlinear simulations predict crane behavior fairly well. The majority of the discrepancy is due to nonlinear frictional effects in the trolley drive system. Therefore, in order to achieve better agreement with the experimental data, a small amount of linear viscous damping was added to the model. The deflection angles of the payload are predicted much better by the nonlinear model than the linear model, as shown by the plots of radial swing in Figure 4 and tangential swing in Figure 5.

2.3 Traditional Input Shapers Applied to Tower Cranes

In order to reduce payload oscillation, this paper presents new command shapers designed to accommodate the nonlinearities associated with rotational motions of tower cranes. Note that purely radial motions of a tower crane have very linear dynamics, so they are not considered here.

A traditional ZV shaper convolved with a slewing profile will yield two accelerations in the tangential direction. Assume that the two accelerations induce oscillations that can be represented by the arrows labeled $A_1$ and $A_2$ in Figure 6. Due to the rotational nature of the tower crane, these accelerations are not in the same direction. The second acceleration is rotated through an angle of $\Delta s$, which is the change in angle that occurs during the time between the two accelerations. Given this effect, the vibration induced by the second acceleration will not completely cancel the vibration induced by the first acceleration, and therefore, the effectiveness of the ZV shaper is decreased.

2.4 Radial-Motion Assisted Command Shapers

In order to better cancel the oscillations induced during a rotational motion, the desired rotational motion can be augmented with a small radial motion. Radial-
motion assisted command shapers that combine these motions are designed to force the system to its steady-state condition, while assuming the $\phi$ and $\theta$ states are uncoupled. This technique will be explained further in the following paragraphs. The radial-motion assisted shaper builds on the relationship between input shaping and the system steady state. First, this relationship will be established for a well-documented problem, ZV shaping a planar crane. Then, these ideas will be applied to the tower crane. Another type of shaper, the Unity Magnitude Zero Vibration (UM-ZV) shaper (Singhose et al., 1994, 1997a) will also be considered in the analysis.

Consider the simple problem of ZV shaping constant-velocity motion of a planar crane. The shaped velocity command consists of two steps; the first step is to half-speed, and the second is to full speed. A ZV shaper accelerates the planar crane up to its steady-state velocity without residual oscillation. Under this condition, the trolley and payload move at a constant, and equal velocity, $v_f$.

Figure 7(a) shows a planar crane response to a half-speed step command. The solid line is the trolley position, and the dashed line is the payload position. When the payload reaches the point labeled “S” its velocity is equal to the full-speed velocity of the trolley and the deflection is zero. Therefore, if the trolley speed were to suddenly increase to full-speed at point “S”, then the system would be at steady state and there would be no vibration. This second speed increase is exactly what the ZV shaper commands the trolley to do. Figure 7(b) shows the full ZV-shaped command and response. As predicted, the trolley velocity switches to full speed at precisely the point “S” when the payload is at the steady-state condition.

Based on these observations, a ZV shaper can be interpreted to work as follows: the shaper moves the system initially, waits for the system to reach a critical point, then changes the command so the system “relaxes” to steady-state. This idea is very old, and has its roots in “Posi-cast” control developed by O.J.M. Smith in the late 1950s (Smith, 1957). A similar approach was used by Smith, et. al. for a system with a linearly-varying natural frequency (Smith et al., 2002). The term “steady-state relaxation” will be used to refer to this strategy.

The same steady-state relaxation approach can be applied to tower cranes, with a few modifications. First, the equilibrium condition of the tower crane for a constant slewing velocity must be established. To find the steady-state deflection of the system, it is assumed that the tower crane is rotating at constant angular velocity and with zero radial motion: $\dot{s} = s_{ss}, \ddot{s} = \ddot{R} = \dot{R} = 0$. The conditions for steady state are zero velocity and acceleration of the deflection angles: $\dot{\phi} = \dot{\theta} = \ddot{\phi} = \ddot{\theta} = 0$. Substituting these assumptions into the nonlinear equations of motion (5) yields:

$$g \sin \theta_{ss} = L \ddot{s}_{ss}^2 \sin \theta_{ss} \cos \phi_{ss} \cos \theta_{ss}$$

(7)

$$g \sin \phi_{ss} \cos \theta_{ss} = R \ddot{s}_{ss}^2 \cos \phi_{ss} + L \ddot{s}_{ss}^2 \sin \phi_{ss} \cos^2 \theta_{ss} \cos \phi_{ss}$$

(8)

where $\theta_{ss}$ and $\phi_{ss}$ are the steady-state deflection angles.
To satisfy (7), set the tangential deflection to zero:

$$\theta_{ss} = 0$$  \hspace{1cm} (9)$$

Substituting this into (8) yields:

$$0 = x_1 \cos(\phi_{ss}) + x_2 \sin(\phi_{ss}) \cos(\phi_{ss}) - \sin(\phi_{ss})$$  \hspace{1cm} (10)$$

where,

$$x_1 = \left( \frac{R \dot{s}_{ss}^2}{g} \right) \hspace{1cm} x_2 = \left( \frac{L \dot{s}_{ss}^2}{g} \right)$$  \hspace{1cm} (11)$$

For small radial deflection angles, $\phi_{ss} < 0.3$, the solution can be approximated as:

$$\phi_{ss} = \frac{x_1}{1 - x_2} = \frac{R \dot{s}_{ss}^2}{g - L \dot{s}_{ss}^2}$$  \hspace{1cm} (12)$$

To summarize, the steady-state conditions of the rotating tower crane are given by $\theta_{ss} = 0$ and $\phi_{ss} = \frac{x_1}{1 - x_2}$ (for small angles). Physically, this corresponds to the payload deflected radially outward in the plane formed by the mast and jib. According to the linearized approximation given in (12), as the slewing speed ($\dot{s}_{ss}$), trolley radial position ($R$), or suspension length ($L$) increase, the steady-state radial deflection ($\phi_{ss}$) will also increase. However, the steady-state tangential deflection ($\theta_{ss}$) is unaffected by any of these parameters.

Figure 8 shows an overhead view of this steady-state condition. Both the trolley and payload have the same instantaneous angular velocity, $\dot{s}_f$. The steady-state payload deflection, $\Delta_{ss}$, is related to the steady-state angle given in (12) by: $\Delta_{ss} = L \sin \phi_{ss}$. The steady-state radial position of the payload, $R_{ss}$, is therefore given by:

$$R_{ss} = R + \Delta_{ss} = R + L \sin \phi_{ss}$$  \hspace{1cm} (13)$$

where $R$ is the radial position of the trolley.

A three-stage process is used to form a vibration-limiting slewing command that is assisted by a radial trolley motion. In the first stage, a ZV-shaped slewing command is used. Figure 9(a) shows the trolley and payload angular position and Figure 9(b) shows the trolley and payload radial position along with the instantaneous steady-state radial position, $R_{ss}$. Note that the payload deflection in the angular direction is the tangential deflection angle, $\theta$.

The ZV shaper appears to work well for the angular response and resembles the response of the planar crane shown earlier in Figure 7(b). However, the ZV command induces oscillation in the radial direction. When the slewing velocity reaches full speed, $R_{ss}$ jumps to a new position. As a result, the payload is no longer at steady
state and it oscillates about this new value of $R_{ss}$. The purpose of this simulation is to identify the time and amplitude of the first peak in the radial response, labeled P in Figure 9(b). This information will be used in the second stage to eliminate the vibration.

The second stage of forming the radial-motion assisted command is to eliminate the residual vibration using the steady-state relaxation concept. In this case, the first peak, P, plays the role of the steady-state point, S, shown earlier in Figure 7(a) for the planar crane. Point P satisfies the steady-state condition of zero velocity, but the position does not equal $R_{ss}$. However, $R_{ss}$ can be adjusted by moving the trolley in the radial direction. More specifically, (13) can predict the trolley radial position that makes $R_{ss}$ equal to P. The trolley is then commanded to quickly move to this new position at the time when the payload reaches point P. Figure 10 shows the resulting response. The payload no longer oscillates because it is at the steady-state radius, $R_{ss}$.

The above command brings the system from rest to a constant slew-velocity. In the third stage, the above process is reversed so that the tower crane returns to rest. An example of a complete command is shown in Figure 11. The system was commanded to start decelerating at $t = 3s$. Note that the command is not symmetric. Also note that the command duration does not need to be known ahead of time. The radial-motion assisted shapers will be denoted with an $R$ subscript. For example, a radial-assisted ZV shaper will be denoted as $ZV_R$.

2.4.1 Advantages and Disadvantages of Radial-Motion Assisted Shapers

There are several advantages and disadvantages to the above technique. The advantages are:

- It compensates for many of the non-linear aspects of a tower crane.
- The technique is guaranteed to do better than standard input shaping.
- The technique can be easily expanded to form different types of commands. In the above case a ZV shaper was used for the slewing velocity, but any other shaper could be used.
- Shapers are faster than other types of traditional robust input shapers such as the ZVD shaper.

The disadvantages are:

- Simulating the system response is required for each set of operating parameters.
- The process assumes that the slewing and radial oscillations are essentially uncoupled. It has already been shown that this is not strictly the case, which is why this technique will always produce some small residual swing.
- Because the command is not symmetric, the final radial position is never exactly the same as the initial radial position. However, in reasonable cases this
difference is very small. The maximum measured change between the initial and final trolley radius is 2% for the crane parameter ranges: \( R = [0.5 \ldots 1] \) m, \( L = [0.5 \ldots 1.5] \) m, and \( \dot{s} = [6 \ldots 30] \) deg/s. It was observed that the percent change in trolley radius increases with the suspension length, \( L \), and slew velocity, \( \dot{s} \). However, the initial trolley radius, \( R \), has a negligible effect on the percent change in the final trolley radial position.

- This technique will not work as planned for very short duration moves. For example, referring to Figure 10(b), the system cannot be commanded to stop before the time of point P if the algorithm is to work as required. However, very short moves induce only small levels of oscillation, so this disadvantage is relatively minor.

As mentioned above, this technique is very flexible. The three-stage process discussed earlier can be generalized as follows:

1. Simulate the system using an input-shaped command for the slewing axis, where the shaper can be any linear input shaper.
2. Measure the time and amplitude of the first peak of the radial swing. Then, generate a step in the trolley radial position such that the steady-state payload radius, \( R_{ss} \), coincides with the value of the first peak at the measured time.
3. Repeat the process to generate the command that returns the system to rest.

### 2.5 Directional Command Shapers

This section presents another type of radial-motion assisted shaper that is based on a different design approach. Rather than trying to achieve “steady-state relaxation,” this class of shapers is based on directional effects. It was noted before that traditional input shaping yields accelerations in different directions when applied to a tower crane. In order to better cancel oscillations, all of the accelerations can be made to act in the same direction by adding radial motion to the rotational motion. The directional ZV shaper, \( ZV_{\text{Dir}} \), is illustrated in Figure 12.

To design this shaper, the resultant vectors from the radial and angular moves, shown as \( \vec{V}_1 \) and \( \vec{V}_2 \), must be set equal. This shaper consists of two sets of impulses, one for angular motions, and one for radial motions. This is represented as:

\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
\gamma_1 & \gamma_2 \\
0 & T/2
\end{bmatrix},
\begin{bmatrix}
B_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
-\delta_1 & \delta_1 \\
0 & T/2
\end{bmatrix}
\]

where \( A_i \) are the impulse amplitudes in the angular direction, \( B_i \) are those in the radial direction, and \( t_i \) are the impulse times. Setting \( \vec{V}_1 = \vec{V}_2 \) and constraining
\[ \sum A_i = 1 \text{ yields:} \]

\[ \gamma_1 + \gamma_2 = 1, \quad \delta_1 = \alpha \gamma_1 R_1, \quad \gamma_2 \left( R_1 - \frac{\delta_1 \dot{s}_r}{2} (T - t_r) \right) = \beta \gamma_1 R_1 \quad (15) \]

where \( R_1 \) is the starting radial position of the trolley, \( \dot{s} \) is the acceleration of an assumed trapezoidal velocity profile with rise time, \( t_r < T/2 \), and:

\[ \alpha = \frac{1}{[1 + \cos(\Delta s)] \cot(\Delta s) + \sin(\Delta s)}, \quad \beta = \alpha \frac{1 + \cos(\Delta s)}{\sin(\Delta s)}, \quad \Delta s = \frac{\gamma_1 \dot{s}_r}{2} (T - t_r) \quad (16) \]

The design parameters (inputs) are the desired slew angle, \( \Delta s \), and \( \dot{s} \). Once these are determined, these equations can easily be solved for the three unknowns, \( \gamma_1 \), \( \gamma_2 \), and \( \delta_1 \), using a standard numerical solver. When the shapers are determined, they are both convolved with the angular velocity profile to obtain shaped velocity profiles for the radial and angular directions. The \( ZV_{\text{Dir}} \) shaper does not have the short-move-duration limitations of the radial-motion assisted shaper based on the steady-state relaxation technique described in the previous section. It also has a simpler design process that requires only knowledge of the system parameters, not simulated payload responses. However, it does require that the slew angle, \( \Delta s \), be known prior to initiation of the motion.

2.6 Simulation Studies

Multiple simulations were used to compare the strengths and weaknesses of the shapers presented above. Six different shapers were tested and they can be divided into two categories:

**Standard Shapers: ZV, UM-ZV, ZVD.** These shapers are called “standard” because they come from basic input-shaping theory for linear systems. For the tower crane simulations, the ZV, UM-ZV, and ZVD shapers were designed for an undamped frequency of \( \sqrt{g/L} \).

**Radial-Motion Assisted Shapers: ZV_R, UM-ZV_R, ZV_{Dir}.** These shapers use combined angular and radial motion.

The shapers were compared using two different performance measures, residual vibration reduction and shaper duration, a measure of the amount of time it takes for the shaped command to reach the commanded velocity. The nominal system parameters used for the simulations are given in Table 1.

2.6.1 Residual Vibration

The residual vibration of each trial was defined as the maximum payload deflection at the completion of the trolley motion. Figure 13 shows the residual vibration
amplitude of shaped and unshaped commands for various pulse commands defined as:

\[ \dot{s}(t) = \dot{s}H(t) - \dot{s}H(t - t_p) \]  

where \( \dot{s} \) is the amplitude of the command, \( t_p \) is the pulse duration, and \( H(t) \) is the heaviside step function.

Figure 13(a) compares the unshaped and ZV shaped residual vibration. Notice that even with standard ZV shaping there is a substantial (90%) reduction in vibration. Figure 13(b) shows the residual vibration for all of the shaped commands. The average reduction in vibration for each shaper compared to the unshaped motion is shown in Table 2.

The performance of the shapers can be ordered from best to worst, as shown in Figure 14. Shapers that share the same cell in Figure 14 have approximately the same vibration-reducing effect. Note that the radial-motion assisted shapers outperform standard ZV and UM-ZV shaping.

Further simulations were performed to compare the performance as a function of slew angle. The results were determined for slewing rotations of 60° through 240°. Figure 15 shows the performance of the three shapers compared to the unshaped case for \( R_0 = 0.55 \text{m} \) and \( L_0 = 1 \text{m} \). The results shown in Figure 15(a) are for a slewing acceleration of \( \ddot{s} = 40 \text{ deg/s}^2 \) and a rise time of \( t_r = 0.5 \text{s} \). The rise time is defined as the amount of time it takes for the jib to slew from zero to full angular velocity for a given angular acceleration. In Figure 15(b) the slewing acceleration was increased to \( \ddot{s} = 120 \text{ deg/s}^2 \), while the rise time was kept the same. It is evident that any form of input shaping is a vast improvement over the unshaped cases for nearly all move distances, even for very aggressive moves.

2.6.2 Experimental Verification

Experimental tests were performed on a portable tower crane at the Tokyo Institute of Technology (Lawrence et al., 2006). The crane was commanded remotely from Atlanta, Georgia using an internet-based control system. The hardware specifications of the crane are similar to those in Table 1. Due to its remote location, the oscillation of the crane could not be completely zeroed out prior to each test. Therefore, the initial swing of the payload for all experiments was approximately 0.02 deg.

The experimental results for the ZV \(_R\) shaper are shown in Figure 16. The vibration in the radial direction is plotted against the slew pulse duration, \( t_p \). In Figure 16(a), the radial position of the trolley, \( R \), is 0.55m and the payload suspension length, \( L \), is 1.0m. The slewing velocity was \( \dot{s} = 20 \text{ deg/s} \). In Figure 16(b), the suspension length was increased to \( L=1.5 \text{m} \). In terms of residual vibration, the ZV \(_R\) shaper is on average 44% better than the ZVD and 117% better than the ZV. In terms of
shaper duration, the ZV and ZV\textsubscript{R} shapers are twice as fast as the ZVD. Considering both these performance criteria, the ZV\textsubscript{R} shaper outperforms both the standard ZV and ZVD shapers.

The experimental results for the ZV\textsubscript{Dir} shaper are shown in Figures 17 and 18. The ZV\textsubscript{Dir} shaper does not perform as well as the ZV\textsubscript{R}, especially in the radial direction. This performance degradation occurs because there is moderate damping in the radial direction of the portable crane. Furthermore, the hardware cannot perfectly track the desired command. The radial velocity profiles have very small magnitudes, magnifying the error signal with respect to the desired velocity. Typical radial velocity errors seen in the experiments were ±14%.

To further investigate the influence of velocity tracking error on residual vibration, the system rise time was increased from $t_r = 0.5\, s$ to $t_r = 1.25\, s$, while keeping the acceleration constant. This increases the maximum slewing velocity, $\dot{s}_{\text{max}}$, from 20 deg/s to 50 deg/s. Figure 19 shows the simulation results for $R_0 = 0.9\, m$ and $L_0 = 1\, m$. Even for these high speeds, input shaping is still able to drastically reduce the residual vibration. The ZV\textsubscript{Dir} shaper performance is in between that of the ZV and ZVD shapers. Figure 20 shows the corresponding experimental results. At these higher velocities, the relative error in the radial velocity is reduced, and the performance of the ZV\textsubscript{Dir} shaper falls in between ZV and ZVD shaping, as expected.

3 Conclusions

Nonlinear equations of motion for a tower crane were presented and verified experimentally. Two novel command-shaping algorithms were presented for slewing tower cranes. The techniques were shown to be more effective than traditional input shaping for reducing vibration in the direction of slewing motions. They were shown to outperform ZV and UM-ZV input shapers, but not the ZVD shaper, in terms of vibration reduction. However, the new command shapers provide faster motion than ZVD shapers. Experimental results from a tower crane operated tele-robotically verified the effectiveness of the new command-shaping techniques.

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References


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Table 1
Nominal System Parameters.

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<td>(m/s$^2$)</td>
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Table 2
Average Vibration Reduction Compared to Unshaped Motion.

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<th>UM-ZV</th>
<th>ZVD</th>
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<td>89.47%</td>
<td>87.70%</td>
<td>96.36%</td>
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<table>
<thead>
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<th>% Vibration Reduction</th>
<th>ZV$_R$</th>
<th>UM-ZV$_r$</th>
<th>ZV$_{\text{Dir}}$</th>
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<tr>
<td></td>
<td>94.39%</td>
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Fig. 1. Input Shaping Process.

![Input Shaping Process](image)

Fig. 2. Tower crane schematic.

![Tower crane schematic](image)

Fig. 3. Experimental and nonlinearly simulated payload trajectories.

![Experimental and nonlinearly simulated payload trajectories](image)
Fig. 4. Experimental and simulated radial swing angles ($\phi$).

Fig. 5. Experimental and simulated tangential swing angles ($\theta$).

Fig. 6. Impulsive Effects of a ZV Shaper.

Fig. 7. Relating the Steady State of the Crane to ZV shaping.
Fig. 8. Overhead View of Tower Crane Steady-State Condition.
Fig. 9. Radial-Motion Assist Shaper Formation: First Stage.

(a) Angular Position

(b) Radial Position

Fig. 10. Radial-Motion Assist Shaper Formation: Second Stage.

(a) Angular Position

(b) Radial Position

Fig. 11. Radial-Motion Assist Shaper Formation: Third Stage.

(a) Angular Position

(b) Radial Position
Fig. 12. Directional Zero-Vibration ($ZV_{\text{Dir}}$) shaper.

(a) Unshaped and ZV Residual Vibration  (b) Residual Vibration of Various Shapers.

Fig. 13. Simulated Residual Vibration for Various Pulse Times.

<table>
<thead>
<tr>
<th>best</th>
<th>worst</th>
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<tbody>
<tr>
<td>$ZV_R$</td>
<td>$ZV_{\text{Dir}}$</td>
</tr>
<tr>
<td>$ZVD$</td>
<td>$UM-ZV_R$</td>
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<tr>
<td>$ZV$</td>
<td>$UM-ZV$</td>
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Fig. 14. Comparison of Shaper Oscillation Suppression
Fig. 15. Simulated Unshaped and Shaped Residual Vibration.

Fig. 16. Experimental Residual Vibration in the Radial Direction.
Fig. 17. Experimental Residual Vibration in the Tangential Direction.
Fig. 18. Experimental Residual Vibration in the Radial Direction.

(a) $R_0 = 0.55$ m, $L_0 = 1$ m
(b) $R_0 = 0.55$ m, $L_0 = 1.5$ m
(c) $R_0 = 0.9$ m, $L_0 = 1$ m
(d) $R_0 = 0.9$ m, $L_0 = 1.5$ m

Fig. 19. Simulated Residual Vibration, $\ddot{s} = 40$ deg/s$^2$, $t_r = 1.25$ s.
Fig. 20. Experimental Residual Vibration with Increased Rise Time.