Friction Force Balance – Determination of \( \mu \)

Overview:
This handout will outline a method, and the theory behind the method, for identifying the coefficient of static friction for a surface. The method outlined utilizes a force balance method around a static equilibrium. A limiting condition on this equilibrium allows for the estimation of the coefficient of static friction.

The Theory:
A block of mass \( m \) is placed on an inclined place at angle \( \theta \), as shown in Figure 1. The

![Figure 1: Schematic of a Block on an Inclined Plane](image)

coefficient of friction of the plane is unknown but is denoted as \( \mu \). If the free body diagram of the block is drawn, shown in Figure 2, one can see that there are three forces acting on the body, the force from its weight, \( mg \), the normal force, \( N \), and the force of friction, \( f \). It is useful to define a new coordinate frame, in addition to the usual X-Y frame, where X is horizontal and Y is vertical. We define a coordinate system that has one component parallel to the plane, which we will call the \( i \)-component, and another perpendicular to it, which we will call the \( j \)-component. This coordinate frame is shown in both Figures 1 and 2.
Because we know that Newton’s Second Law holds in all directions, the forces and accelerations in the additional frame can be related. The advantage of this additional frame is that it allows us to express the acceleration of the block as an acceleration in only one direction, down the plane. However, the force from the weight of the object must be resolved into the new coordinate frame. After doing this, Newton’s Second Law can be written for each direction.

\[ \sum F = (f - mg \sin \theta)i + (N - mg \cos \theta)j \]

Knowing that there is no motion in the \( j \)-direction, that component of the net forces can be set to zero.

\[ N - mg \cos \theta = 0 \]

We can then solve for the normal force.

\[ N = mg \cos \theta \]

This allows us to write the maximum force that friction is able to provide.

\[ f_{\text{max}} = \mu N = \mu mg \cos \theta \]

At the instant before the block begins to slip, the frictional force reaches its maximum while the block is still motionless. Using this fact, we can write the force balance in the \( i \)-direction.

\[ f_{\text{max}} - mg \sin \theta = \mu mg \cos \theta - mg \sin \theta = 0 \]

Now we can solve for \( \mu \) to find,
\[ \mu mg \cos \theta = mg \sin \theta \]
\[ \mu \cos \theta = \sin \theta \]
\[ \mu = \frac{\sin \theta}{\cos \theta} = \tan \theta. \]  \hspace{1cm} (6)

**The Experimental Method**

The result from the theoretical work above provides a useful method to determine the coefficient of friction of a surface. First, let’s examine Equation (6). We see that for an inclined plane the coefficient of friction can be determined solely from the angle of incline at which slipping first occurs. The mass of the object has no effect on this determination. So, we can place a block on the surface to be tested and slowly increase the angle until slipping occurs. Using this angle, we can determine the coefficient of friction of the surface.